Amendments to the Specification

Beginning on page 6, line 11 to page 8, line 5, please replace the equations with the following more legible equations:

Equations:

$$\frac{\partial}{\partial t}\left(rw\sqrt{1+\left(\frac{\partial r}{\partial z}\right)^2}\right)+\frac{\partial}{\partial z}(rwv)=0, \tag{1}$$

Where,

$$t=\frac{\bar{t}\bar{v}_0}{\bar{r}_0},\quad z=\frac{\bar{z}}{\bar{r}_0},\quad r=\frac{\bar{r}}{\bar{r}_0},\quad v=\frac{\bar{v}}{\bar{v}_0},\quad w=\frac{\bar{w}}{\bar{w}_0}$$

Axial direction:

$$\frac{2rw[(\tau_{11}-\tau_{22})]+2r\sigma_{surf}}{\sqrt{1+(\partial r/\partial z)^2}}+B(r_F^2-r^2)-$$
$$2C_{gr}\int_0^{2_L} rw\sqrt{1+(\partial r/\partial z)^2}\,dz-2\int_0^{2_L} rT_{drag}d_z=T_z \tag{2}$$

Where,

$$T_z = \frac{\overline{T_z}}{2\pi\eta_0 \overline{w_0}\,\overline{v_0}}, \quad B = \frac{\overline{r_0^2}\,\Delta P}{2\eta_0 \overline{w_0}\,\overline{v_0}},$$

$$\Delta P = \frac{A}{\int_0^{z_L} \pi \overline{r}^2 \,\mathrm{d}\overline{z}} - P_a, \quad \tau_{ij} = \frac{\overline{\tau_{ij}}\,\overline{r_0}}{2\eta_0 \overline{v_0}}$$

$$C_{gr} = \frac{\rho g \overline{r_0^2}}{2\eta_0 \overline{v_0}}, T_{drag} = \frac{\overline{T_{drag}}\,\overline{r_0^2}}{2\eta_0 \overline{v_0}\,\overline{w_0}}, \sigma_{surf} = \frac{\overline{\sigma_{surf}\, r_0}}{2\eta_0 \overline{v_0}\,\overline{w_0}}$$

Circumferential direction:

$$B = \left(\frac{\left[-w(\tau_{11}-\tau_{22})+2\sigma_{surf}\right](\partial^2 r/\partial z^2)}{\left[1+(\partial r/\partial z)^2\right]^{3/2}} + \frac{w(\tau_{33}-\tau_{22})+2\sigma_{surf}}{r\sqrt{1+(\partial r/\partial z)^2}} - C_{gr}\frac{\partial r/\partial z}{\sqrt{1+(\partial r/\partial z)^2}} \right)$$

(3)

Constitutive Equation:

$$K\boldsymbol{\tau} + De\left(\frac{\partial \boldsymbol{\tau}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{\tau} - \boldsymbol{L}\cdot\boldsymbol{\tau} - \boldsymbol{\tau}\cdot\boldsymbol{L}^{\mathrm{T}}\right) = 2\frac{De}{De_0}\boldsymbol{D}, \tag{4}$$

where $K = \exp[\varepsilon\, De\, \mathrm{tr}\, \boldsymbol{\tau}]$, $\boldsymbol{L} = \nabla\boldsymbol{v} - \xi\boldsymbol{D}$, $2\boldsymbol{D} = (\nabla\nu +$ $\nabla\boldsymbol{v}\mathrm{T})$, $De_0 = \frac{\lambda\bar{\boldsymbol{v}}_0}{\bar{r}_0}$, $De = De_0\ \exp\left[k\left(\frac{1}{\theta} - 1\right)\right]$.

Energy Equation:

$$\frac{\partial\theta}{\partial t} + \frac{\nu}{\sqrt{1+(\partial r/\partial z)^2}}\frac{\partial\theta}{\partial z} + \frac{U}{w}(\theta - \theta_{\mathrm{c}}) + \frac{E}{w}(\theta^4 - \theta_\infty^4) = 0, \tag{5}$$

Where,

$$\theta = \frac{\bar{\theta}}{\theta_0}, \quad \theta_{\mathrm{c}} = \frac{\bar{\theta}_{\mathrm{c}}}{\theta_0}, \quad \theta_\infty = \frac{\bar{\theta}_\infty}{\theta_0}, \quad U = \frac{\bar{U}\bar{r}_0}{\rho C_P \bar{w}_0 \bar{\nu}_0},$$

$$E = \frac{\varepsilon_{\mathrm{m}}\sigma_{\mathrm{SB}}\bar{\theta}_0^4\bar{r}_0}{\rho C_P \bar{w}_0 \bar{\nu}_0 \theta_0}$$

Boundary conditions:

$$\nu = w = r = \theta = 1, \qquad \tau = \tau_0 \quad \text{at } z = z_0, \tag{6a}$$

$$\frac{\partial r}{\partial t} + \frac{\partial r}{\partial z}\frac{\nu}{\sqrt{1+(\partial r/\partial z)^2}} = 0, \qquad \frac{\nu}{\sqrt{1+(\partial r/\partial z)^2}} = D_R,$$

$$\theta = \theta_F \quad \text{at } z = z_F. \tag{6b}$$

Beginning on page 12, line 13 to page 14, line 6, please replace the equations with the following more legible equations:

Equations:

$$\frac{\partial}{\partial t}\left(rw\sqrt{1+\left(\frac{\partial r}{\partial z}\right)^2}\right) + \frac{\partial}{\partial z}(rw\nu) = 0, \tag{1}$$

Where,

$$t = \frac{\bar{t}\bar{\nu}_0}{\bar{r}_0}, \quad z = \frac{\bar{z}}{\bar{r}_0}, \quad r = \frac{\bar{r}}{\bar{r}_0}, \quad \nu = \frac{\bar{\nu}}{\bar{\nu}_0}, \quad w = \frac{\bar{w}}{\bar{w}_0}$$

Axial direction:

$$\frac{2rw[(\tau_{11}-\tau_{22})]+2r\sigma_{surf}}{\sqrt{1+(\partial r/\partial z)^2}}+B(r_F^2-r^2)-$$
$$2C_{gr}\int_0^{z_L} rw\sqrt{1+(\partial r/\partial z)^2}\,dz-2\int_0^{z_L} rT_{drag}d_z=T_z \qquad (2)$$

Where,

$$T_z=\frac{\overline{T_z}}{2\pi\eta_0\overline{w_0}\,\overline{v_0}},\quad B=\frac{\overline{r_0^2}\,\Delta P}{2\eta_0\overline{w_0}\,\overline{v_0}},$$

$$\Delta P=\frac{A}{\int_0^{z_L}\pi\overline{r}^2\,d\overline{z}}-P_a,\quad \tau_{ij}=\frac{\overline{\tau_{ij}}\,\overline{r_0}}{2\eta_0\overline{v_0}}$$

$$C_{gr}=\frac{\rho g\overline{r_0^2}}{2\eta_0\overline{v_0}},T_{drag}=\frac{\overline{T_{drag}}\,\overline{r_0^2}}{2\eta_0\overline{v_0}\,\overline{w_0}},\sigma_{surf}=\frac{\overline{\sigma_{surf}r_0}}{2\eta_0\overline{v_0}\,\overline{w_0}}$$

Circumferential direction:

$$B=\left(\frac{\left[-w\left(\tau_{11}-\tau_{22}\right)+2\sigma_{surf}\right]\left(\partial^2 r/\partial z^2\right)}{\left[1+\left(\partial r/\partial z\right)^2\right]^{3/2}}+\frac{w\left(\tau_{33}-\tau_{22}\right)+2\sigma_{surf}}{r\sqrt{1+\left(\partial r/\partial z\right)^2}}-C_{gr}\frac{\partial r/\partial z}{\sqrt{1+\left(\partial r/\partial z\right)^2}}\right)$$

(3)

Constitutive Equation:

$$K\boldsymbol{\tau}+De\left(\frac{\partial\boldsymbol{\tau}}{\partial t}+\boldsymbol{v}\cdot\nabla\boldsymbol{\tau}-\boldsymbol{L}\cdot\boldsymbol{\tau}-\boldsymbol{\tau}\cdot\boldsymbol{L}^{\mathrm{T}}\right)=2\frac{De}{De_0}\boldsymbol{D},$$

(4)

where $K=\exp[\varepsilon\, De\, \mathrm{tr}\,\boldsymbol{\tau}]$, $\boldsymbol{L}=\nabla\boldsymbol{v}-\xi\boldsymbol{D}$, $2\boldsymbol{D}=(\nabla\nu+\nabla\boldsymbol{v}\mathrm{T})$, $De_0=\frac{\lambda\bar{\boldsymbol{v}}_0}{\bar{r}_0}$, $De=De_0\,\exp\left[k\left(\frac{1}{\theta}-1\right)\right]$.

Energy Equation:

$$\frac{\partial\theta}{\partial t}+\frac{\nu}{\sqrt{1+(\partial r/\partial z)^2}}\frac{\partial\theta}{\partial z}+\frac{U}{w}(\theta-\theta_{\mathrm{c}})+\frac{E}{w}(\theta^4-\theta_\infty^4)=0,$$

(5)

Where,

$$\theta = \frac{\bar{\theta}}{\bar{\theta}_0}, \quad \theta_c = \frac{\bar{\theta}_c}{\bar{\theta}_0}, \quad \theta_\infty = \frac{\bar{\theta}_\infty}{\bar{\theta}_0}, \quad U = \frac{\bar{U}\bar{r}_0}{\rho C_P \bar{w}_0 \bar{v}_0},$$

$$E = \frac{\varepsilon_m \sigma_{SB} \bar{\theta}_0^4 \bar{r}_0}{\rho C_P \bar{w}_0 \bar{v}_0 \bar{\theta}_0}$$

Boundary conditions:

$$v = w = r = \theta = 1, \qquad \tau = \tau_0 \quad \text{at } z = z_0, \tag{6a}$$

$$\frac{\partial r}{\partial t} + \frac{\partial r}{\partial z} \frac{v}{\sqrt{1 + (\partial r/\partial z)^2}} = 0, \qquad \frac{v}{\sqrt{1 + (\partial r/\partial z)^2}} = D_R,$$

$$\theta = \theta_F \quad \text{at } z = z_F. \tag{6b}$$